RCS in Radar Range Calculations for Maritime Targets

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1 Abstract

This web page deals with the RCS (Radar Cross Section) parameter and its application in radar range calculations for the detection of maritime targets. It is the intention of the author to compile the basic facts, which are spread in the technical literature and difficult to find, and to comment them where necessary. RCS data of ships quoted even in technical standards, are often incomplete in that their conditions, such as radar frequency, applicable target aspect range, and statistical properties, are missing. These conditions should be taken into account when performing radar range calculations in order to obtain meaningful results. In this paper the basic facts related to RCS shall be elucidated. Where there are open questions yet, these will be mentioned.

2 What is RCS

2.1 Definitions

In radar reference books we find various definitions for RCS, e.g.:

BARTON [1], *"Measure of the reflective strength of a target"*.

The E. W. Handbook of U.S. Navy defines, *"A measure of the radar reflection characteristics of a target. It is equal to the power reflected back to the radar divided by power density of the wave striking the target. For most targets, the radar cross section is the area of the cross section of the sphere that would reflect the same energy back to the radar if the sphere were substituted. RCS of sphere is independent of frequency if operating in the far field region".*

SKOLNIK [2] provides the following short and concise definition, *"The radar cross section of a target is the (fictional) area intercepting that amount of power which, when scattered equally in all directions, produces an echo at the radar equal to that from the target".*

2.2 Physics

Radiation theory teaches us the energy intercepted by an object can be reflected, absorbed, or transmitted through the target. The respective shares of the energy add up to 100 %. With the maritime targets of interest here, we can assume that most of the energy is reflected. RCS, as understood in this paper, shall represent the reflective strength of a radar target.

RCS, denoted by the Greek letter σ and measured in m², is defined as [3]:

$$\sigma = 4\pi \cdot \frac{P_s}{P_i} \qquad \text{Eq. 1}$$

P_i := power density, or intensity, of a plane wave striking the target, (W/m²),
P_s := power per unit solid angle reflected by the target, (W/sr = W).

RCS has a wide spread ranging from 10^{-5} for small insects to 10^{6} for large ships. Hence, RCS is often stated in the logarithmic decibel scale:

$$\sigma_{dBsqm} = 10 \cdot \log\left(\frac{\sigma}{1 \cdot m^2}\right) \qquad \text{Eq. 2}$$

RCS is a function of: [4]

- Position of transmitter/receiver relative to target,
- Target geometry and material composition,
- Angular orientation of target relative to transmitter/receiver,
- Frequency or wavelength,
- Antenna polarisation.

3 The Importance of RCS in Radar Range Calculations

The following formula is used in the author's 'Blanket' algorithm [5] and makes it possible to determine the free space range of a radar system, i.e. the hypothetic maximum radar range.

$$R_{fs} = \left[\frac{P_p \cdot G^2 \cdot \sigma \cdot \lambda^2}{(4\pi)^3 \cdot k \cdot T_0 \cdot B \cdot F_n \cdot (S/N) \cdot L_s}\right]^{1/4} \qquad \text{Eq. 3}$$

The parameters in the formula are predominantly either physical constants or equipment parameters with well defined values, see Table 1 below. In practical applications the two parameters σ and SN have distinct statistical properties so that the calculated range itself inherits statistical properties: the range is related to a certain detection probability.
Both, the theories of handling signals with noise and fluctuating radar targets are well researched since a long time. Common radar range prediction programs, such as CARPET, are taking appropriate care of the two statistical parameters.
In not all range calculation formulae or programs, however, it becomes evident that RCS of fluctuating targets must be handled in a proper way to achieve meaningful data. Eq. 3

above, for instance, S/N must be increased to take care of target echo fluctuations using the appropriate Swerling case [6].

Parameter	**Description**	**Comment**
P_p	Peak pulse power	Equipment parameter
G	Antenna gain	Equipment parameter
σ	Radar cross section of target	*Parameter with large statistical variations for complex radar targets, e.g. a ship*
λ	Wavelength of radar frequency	Equipment parameter
K	Boltzmann's constant	Physical constant
T_0	Absolute temperature of the radar receiver circuitry	Physical variable
B	Bandwidth	Equipment parameter
F_n	Noise figure of the radar receiver	Equipment parameter
S/N	Signal-to-noise ratio required for detection	*Equipment parameter dependent on the desired detection probability for a given false alarm rate*
L_s	System losses	Equipment parameter dependent on the losses of microwave radiation on the path from the transmitter to the antenna and vice versa

Table 1: Radar Range Calculation Parameters

4 RCS of Objects and its determination

SKOLNIK states in his RCS definition that RCS is a ‘fictional’ area. The term ‘area’ refers to the unit being m². ‘Fictional’ means that RCS can actually be much larger than the reflective surface, as the following formula shows:

$$\sigma := A_p \cdot R \cdot D \qquad \text{Eq. } 4$$

Ap	: =	projected object surface,
R	: =	Reflectivity, re-radiated fraction of intercepted power,
D	: =	Directivity, ratio of the maximum intensity of the radiator to the intensity of an isotropic source.

Whereas the reflectivity is usually smaller than unity and material dependent, the directivity can be much larger and depends on the shape of the object.

4.1 RCS of Simple Target Objects

For simple target objects, such as flat rectangular plates, cylinders, spheres, RCS can be calculated using Maxwell’s equations with certain boundary conditions. Figure 1 shows three simple objects with the principal dimension of 1 m and their RCS.

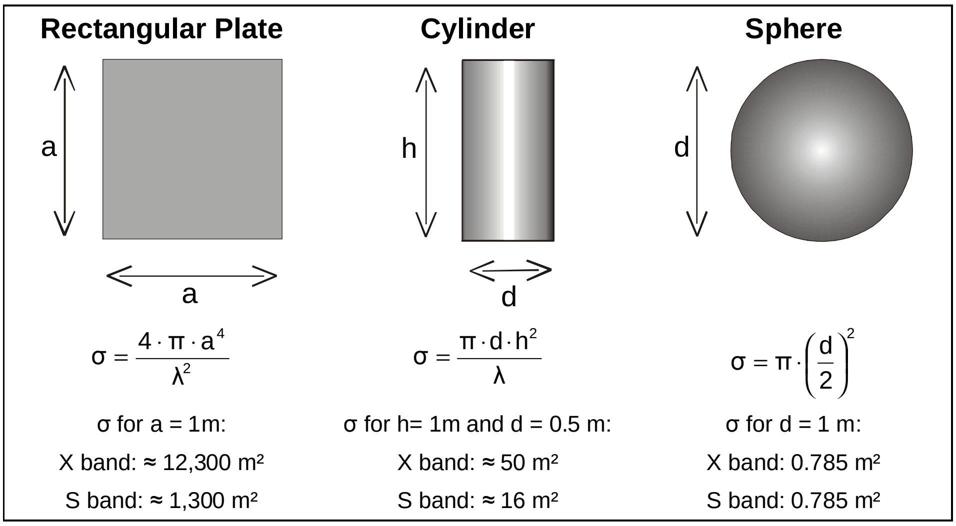


Figure 1: Simple Reflectors

4.2 RCS of Complex Objects and its determination

A complex target is one that consists of several reflectors within a radar resolution cell.

A radar resolution cell is delineated by the radar pulse's length and width of arc in the air.

Following this definition, almost all real-world maritime targets are complex targets. For such targets there is no firm relationship between a target's surface and RCS. Hence, the RCS must be determined in other ways.

An obvious method of determining RCS is to put the object of interest, be it a ship or an aircraft, into a controlled environment and to use a calibrated radar system to measure the echo power. The target's RCS can then be established using the radar range equation taking care of all system parameters and environmental losses. This is basically the procedure performed in so-called 'measuring ranges'. Measurements are usually performed for a 360° aspect arc, at various grazing angles, and often for different radar frequencies.

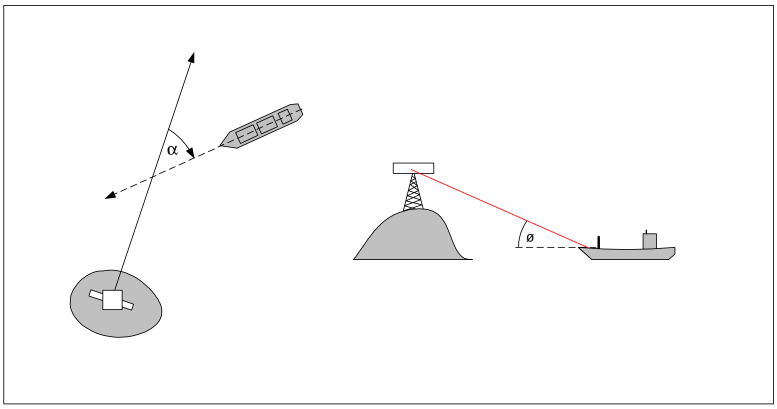


Figure 2: Aspect and grazing angles

"The aspect of a target is its orientation to the axis of the radar beam. … The nearer the angle between the reflecting area and the beam axis is 90°, the greater is the strength of the echo returned to the antenna" [7]. *"Grazing angle is the angle measured in the vertical plane between the ray and a reflecting surface."* [1]

When performing range calculations, the orientation of a target with respect to the radar is often just roughly known. This is, for instance, frequently the case in ship encounters at the open sea. In order to provide a *single* RCS value representing a certain type and size of ship one should define a value with a firm statistical significance [4]. The median value derived from the measurement data set is often used for this purpose. (c.f. chapter 6).

This single RCS value can then be conceived as that of a radar reflector representing the ship. The ideal radar reflector is a sphere, due to its non-directivity and frequency dependence, c.f. RCS definitions in chapter 2.1. This RCS value representing the ship is often accompanied by a height (c.f. chapter 9).

The larger and less mobile a target, the more expensive is the determination of RCS in a measuring range. To save cost, a size-reduced model with appropriately scaled radar frequencies can be used. Another possibility is to simulate the measurements using computer-based methods. By means of construction plans, the target can be decomposed into simple computable reflector elements. The RCS at each aspect angle is then determined by summation of the RCS of the reflecting elements.

5 Statistical Properties of RCS

It has been mentioned above that the echo strength of maritime targets fluctuates a great deal from one echo received to the next. This fluctuation is caused by several effects:

- predominantly random effects, e.g. target scintillation, multipath effects, environmental effects, as caused by atmosphere and seastate,
- systematic effects related to the scattering characteristics, e.g. target strength variations due to aspect and grazing angle changes.

In practice these types of fluctuation can hardly be separated from each other and, hence, are treated statistically in common. If an RCS data set with sufficiently large number of measurements exists, its statistical properties can be determined:

- the RCS mean or median value and
- the shape of the probability density function (PDF),
- the autocorrelation function (ACF).

In 1954 Peter Swerling has published five model cases [6] describing typical radar echo fluctuations. These models can be used in radar range calculations to determine additional S/N margins taking care of the fluctuations [6,2] . In order to determine the applicable Swerling case the PDF and the ACF should be known.

Maritime targets are usually characterised by echoes, which do not fluctuate much from one radar pulse to the next, i.e. successive echoes have significant similarity, whereas echoes from two successive scans are independent from each other (uncorrelated). Such type of fluctuations is described by Swerling case 1, which is characterised by the following PDF:

$$p(\sigma) = \frac{1}{\sigma_{av}} \cdot \exp\left(-\frac{\sigma}{\sigma_{av}}\right) \qquad \text{Eq. 5}$$

σ_{av} := RCS mean value

A Swerling case 1 target is characterised by many scatterers of comparable size. Case 1 is the basic model for most complex scatterers, including maritime targets (ships and radar reflectors).

If a number of radar measurements is available, it is good practice to determine the PDF to ensure that Swerling case 1 applies. Programs for radar range calculation, such as CARPET [8], require the input of the number of the Swerling case in addition to the target's RCS. If the range calculation is performed 'manually', the additional S/N required for a particular Swerling case can be read from a diagram devised by Swerling as a function of the False Alarm Probability [6, 2].

6 The Determination of RCS from a Data Set

As no measurement data set was available to the author, a data set obtained by computer based RCS simulation (CADRCS) [9] was analysed and used to calculate RCS values useful for radar range determination. The data set refers to the research vessel of 50 m length shown in Figure 1.

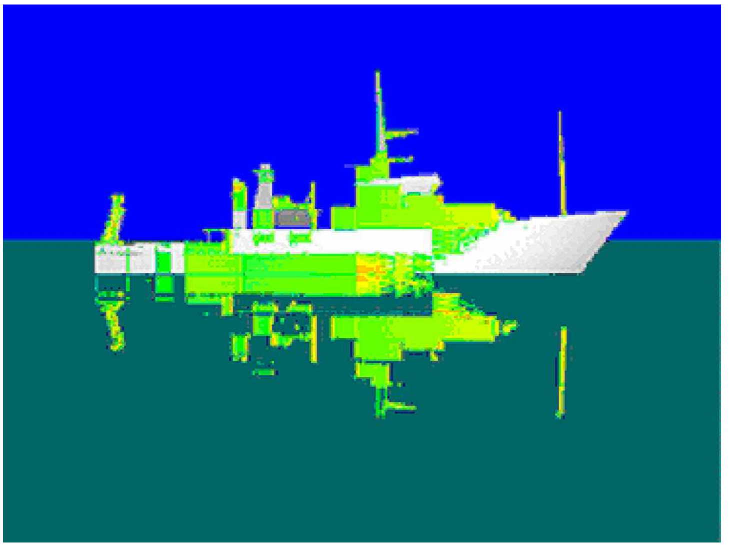


Figure 3: 50 m Research vessel with radar reflection hot spots (Source: CADRCS [9])

As a first step it is useful to visualise the RCS data in form of an x/y plot (Figure 4).

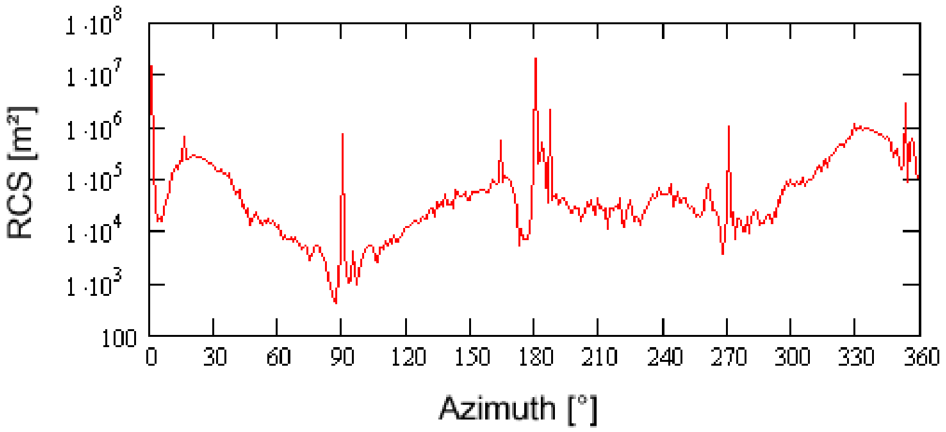


Figure 4: RCS Data set used for RCS data evaluation

RCS data from real measurements are often recorded in decibels rel 1 m², and a polar plot is often used to visualise the data (Figure 5).

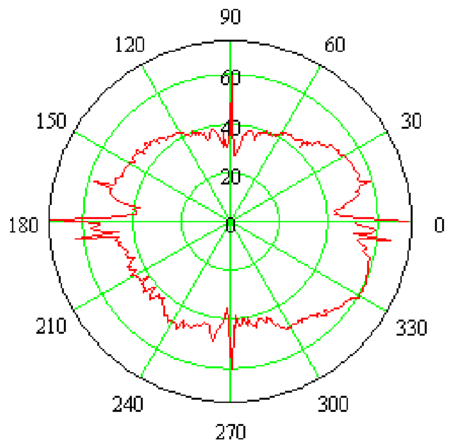


Figure 5: Polar diagram of RCS data in Decibels

The graph of the original RCS data (Figure 4) shows features, which are quite typical for ships. At bow, stern and broadside aspect angles there are ‘highlights’ (very strong specular reflections). In order to avoid an undue influence in the statistics, in the next step they are reduced to an average of their neighbouring values. Figure 6 shows the ‘cleaned’ data set.

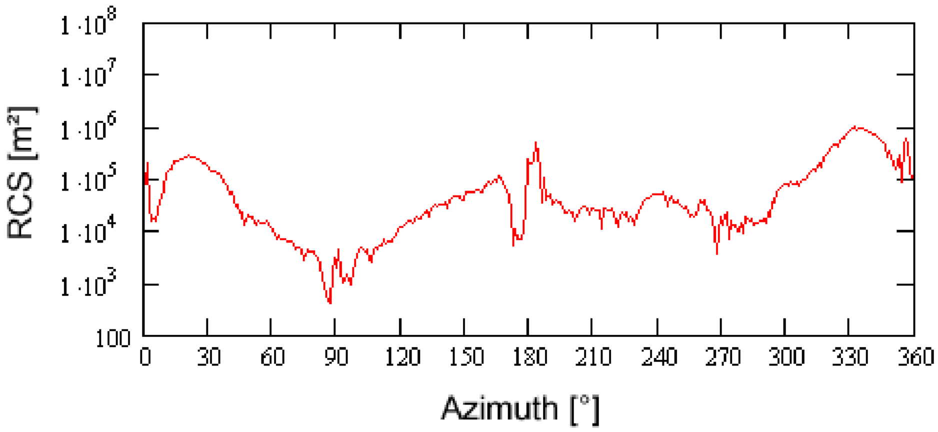


Figure 6: Data set 'cleaned' from 'highlights'

From the clean data set we can calculate the basic statistical parameters (Table 2):

Parameter	RCS [m²]	RCS [dB]
Minimum	422	26.2
Maximum	1,019,108	60.1
Mean	107,369	50.3
Median	29,652	44.7

Table 2: Basic statistical parameters of the 'clean' RCS data set

This simple evaluation shows two distinct characteristics of the data set:

- extremely large data range ($10^2 < \sigma < 10^6$ m²)
- Mean and median RCS deviate from one another, the statistical distribution is not symmetrical.

Using the RCS mean value from Table 2 and assuming that the ship is a Swerling case 1 target we would be ready to perform radar range calculations. It is interesting to note here, that the RCS mean value is considerably larger than those usually stated for ships of similar size (c.f. chapter 9).

As RCS mean and median value differ, we have no a priori knowledge of the probability level of the *mean value* from Table 1, which has to be used with radar range calculation programs, such as CARPET. For this purpose the knowledge of the probability is actually not required, but it must be known when calculating the *detection probability* manually.

As the next step, we shall establish the probability density function (PDF) and the cumulative probability density function (CDF) in order to determine RCS values with defined probabilities.

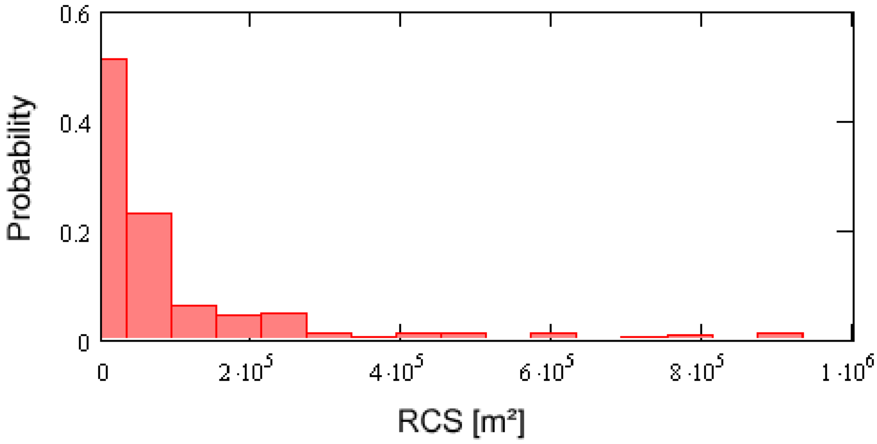


Figure 7: Probability density of the RCS data set

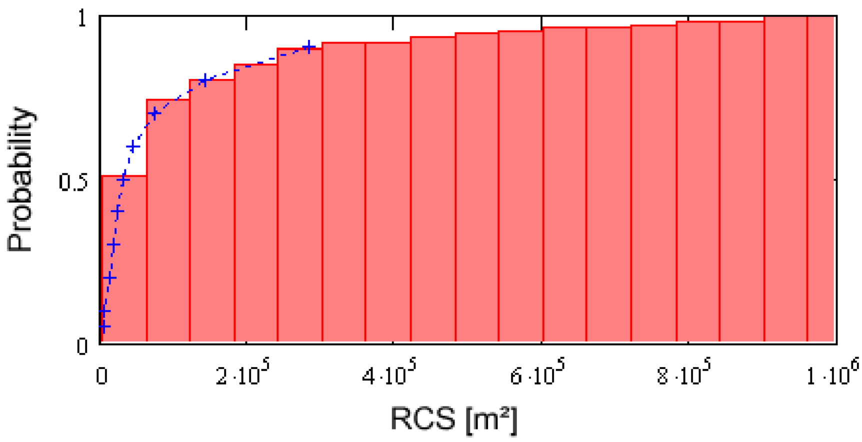


Figure 8: Cumulative Probability of the RCS data

The diagram in Figure 8 provides the probability that the RCS value is smaller than or equal to the value selected on the x axis, whereas Figure 9 provides the RCS value for a given cumulative probability.

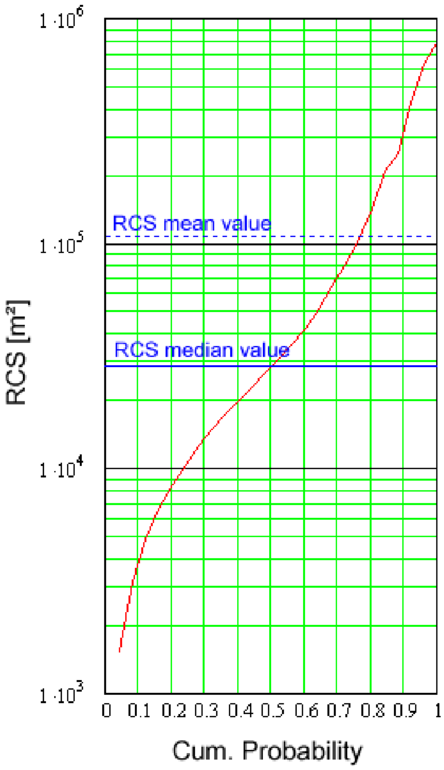


Figure 9: Percentile Plot of RCS

From Figure 9 the - so far unknown - cumulative probability (P) of the RCS mean value can be read. The calculated value is 0.77. When handling radar detection probability one is normally interested in the probability that a target is *not* smaller then a given RCS value (counter probability 1-P). In this case the probability that the RCS is not smaller then 107369 m² is 23%. This seems to be a low probability level. On the other hand, the RCS is quite large. This low probability level will not reduce the detection probability of the radar if the range calculation program requests the RCS mean value *and* applies the Swerling case correctly. For manual range calculation one would usually prefer a higher probability level, e.g. 90% (1-P), and work with the respective RCS of 4501 m² (Table 3). The detection probability will then have to be determined separately.

Percentile (P)	Probability (1-P)	Cumulative RCS [m²]
0.05	0.95	2977
0.1	0.9	4501
0.2	0.8	11.027
0.3	0.7	16.208
0.4	0.6	22.253
0.5	0.5	29.652
0.6	0.4	43.931
0.7	0.3	73.472
0.8	0.2	141.858
0.9	0.1	283.697

Table 3: RCS percentiles and probability

6.1 RCS data sub-sets covering ship aspect quadrants

In practice at sea, as well as in shore-based applications (VTS), certain aspect arcs are more likely to occur than others. In encounter/overtaking situations the radar will ‘see’ the bow/stern of the other vessel. In crossing situations one ship will predominantly see the broadside of the other. Similar situations exist with coastal radar stations, were the fairway either leads towards the radar station (bow or stern view), or if the fairway follows the coast line (broadside view - Figure 10).

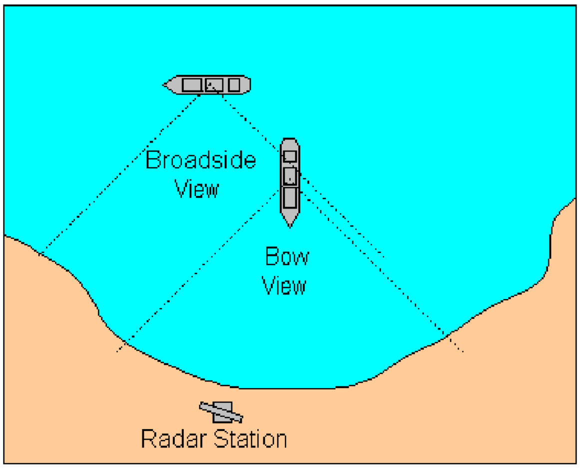
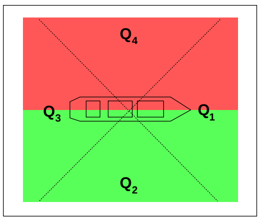


Figure 10: RCS aspect quadrants

Hence, it makes sense to determine RCS data for subsets of the full 360° data set which correspond to aspect arcs that are typical for the radar observation situations mentioned above. The author has examined aspect arcs that correspond to the following ship’s quadrants:



Q_1	Bow view	red 045° to green 045°
Q_2, Q_4	Broadside view	green 045° to green 135° red 045° to red 135°
Q_3	Stern view	green 135° to red 135°

Figure 11: Ship aspect quadrants

The table below show the RCS data for the examined ship aspect quadrants:

Percentile P	Probability 1-P	Bow (Q1)	Broadside (Q2Q4)	Stern (Q3)
0.05	0.95	925	14,935	9,601
0.1	0.9	1,334	21,238	12,409
0.2	0.8	3,087	29,015	16,559
0.3	0.7	4,502	35,645	18,929
0.4	0.6	5,107	52,393	21,938
0.5	0.5	6,292	83,117	28,211
0.6	0.4	7,943	141,149	39,254
0.7	0.3	13,191	220,515	50,144
0.8	0.2	15,216	282,718	71,144
0.9	0.1	18,598	589,357	90,186

Table 4: RCS data of ship quadrants

The RCS values from Table 6 can be used to provide more relevant range calculations whenever certain aspects arcs are dominant in radar observation. The RCS differences between the quadrants are quite large, so that the use of quadrant RCS will provide more relevant range calculations.

7 RCS of Maritime Targets

RCS data of maritime targets are specified by maritime organisations, e.g. IALA [10], by manufacturers of maritime radar systems, by users of coastal radar systems, and in radar reference books. A compilation of the data [11] shows that at least for certain vessel sizes, the RCS data are inconsistent.

A comprehensive study of the RCS of maritime targets has been published by WILLIAMS; CRAMP; CURTIS in 1978 [12]. The reflection properties of buoys, radar reflectors, floating debris and of passing ships have been systematically determined at various distances using a coastal radar station and reference reflectors. The study provides RCS bars for different types of vessel. The bars are annotated with the pertaining aspect sectors. As no newer data seem to be available, the table has been reproduced and amended by estimated figures for the border values of each RSC bar (c.f. Ship RCS Table [15]).

It should be noted that the data in the table are *median* values, and should be adjusted to mean values before using them for range calculations. For Swerling case 1 (Eq. 5) the following relation applies: $\sigma_{mean} \approx 1.44\ \sigma_{median}$.

8 Frequency Dependence of RCS

The specification of a ship's RCS is incomplete, if the radar frequency is missing for which the RCS applies. The simple target forms of Figure 1 have frequency dependencies of f^0, f^1, and f^2, as shown in their RCS computation formulae. For complex targets there is obviously no firm relation between RCS and radar frequency, so that we have to resort to empirical findings.

In his book [2] SKOLNIK provides a formula for the estimation of the RCS of (war)ships in which the ship's displacement and the radar frequency are the variables. This formula suggests $\sigma \sim f^{0.5}$. SKOLNIK also provides two polar RCS plots for a naval vessel measured with X and S band frequencies. From these two plots a frequency dependence of $f^{0.85}$ can be derived, as evaluations of the author show (Figure 12).

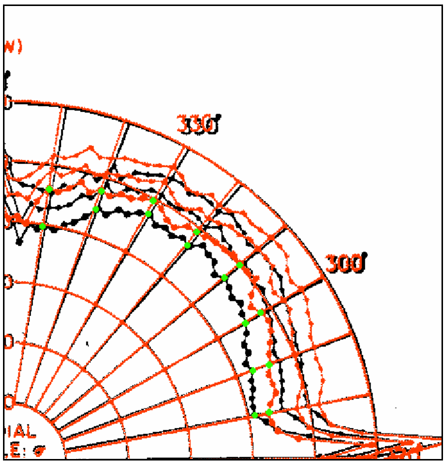


Figure 12: RCS of naval vessel measured with X and S band (based on Skolnik [2])

In 1995 CORENMAN, HAWLEY, HONEY and HONEY undertook on behalf of the U.S. Sailing Association a laboratory test of a number of radar reflectors for onboard use [13]. In their paper they state that RCS for S band is 6 dB smaller than for X band. For the given radar frequencies this is equivalent with $\sigma \sim f^{1.22}$. An evaluation by the author of Table 1 of the tests results shows a somewhat smaller dependence of $\sigma \sim f^{0.9}$ (median value for all tests).

It may be interesting to note that a very similar relation applies for the RCS of the *natural* target of a man. The author's evaluation of the measurement data published already in 1958 by SCHULTZ, BURGENER and KING [14] for the monostatic arrangement and five aspect angles yield $\sigma \sim f^{0.861}$.

For typical X and S band frequencies used in the maritime domain (9.375 and 3.010 GHz) the following relations for f^n apply:

Exponent n	Decibels	(σ_S/σ_X) *100%
2	≈ 10	≈ 10 %
1.2	≈ 6	≈ 25 %
1	≈ 5	≈ 33 %
0.8	≈ 4	≈ 40 %
0.6	≈ 3	≈ 50%

Table 5: Frequency dependence exponents, Decibels and percentages

As a rule of thumb we could use an exponent of 0.8. This means that the X band RCS should be reduced to 40 % to provide an estimate for the S band RCS of the same (complex) target. It should be noted that this relation is not based on a sufficient number of cases and should be used with caution. The validity of the exponential relation is not quite clear. RCS measurements of maritime targets with frequencies in the range of 30 to 100 GHz indicate that it does not apply up to this frequency range.

9 The Target Height Parameter in Maritime Application

Range calculation programs, such as CARPET, use the target height for calculating the range at which the target will disappear behind the radar horizon. When a ship sails away from a radar station, then the obtainable radar range is determined by two physical facts: (i) the target is fully visible, but the echo becomes too weak for detection, or (ii) the echo becomes too weak when the RCS shrinks as target dips on the horizon, whichever occurs first.

The formula for the radar horizon / target visibility range is well known:

$$\frac{\mathbf{R}}{\mathbf{km}} = \mathbf{2.23} \cdot \left(\sqrt{\frac{\mathbf{h_a}}{\mathbf{m}}} + \sqrt{\frac{\mathbf{h_t}}{\mathbf{m}}} \right) \qquad \text{Eq. 6}$$

where h_a = antenna height and h_t := target height.
For $h_t = 0$ we obtain the radar horizon and for $h_t > 0$ the radar target visibility range.

Several sources state the RCS of ships together with height parameters. However, none of the sources available to the author, including the IALA VTS Manual, provide an explanation of the height. The question arises as to whether the height parameter for maritime targets can be used in the same way as the aircraft altitude, extending the range beyond the radar horizon. A pre-requirement for this extension would be that the stated RCS is always located entirely above the stated height. A ship could then dip on the horizon to the amount of the height, while still maintaining its RCS.

At his request, the author was provided with an explanation by IALA, saying that a vessel's RCS can be represented by a radar reflector of given RCS and height. This explanation suggests that the height parameter can be used in the same way as the aircraft altitude.

IALA specifies for an 80 m steel vessel a RCS of 300 m² [10] and a height of 8 m. To clarify the consequences of the height for range calculations, let us take the ship shown in Figure 13 as an example. With its length of 82.5 m it should come close to the IALA 80 m steel vessel.

If the radar reflector (h = 8 m) said to represent the vessel were just to start dipping on the horizon, then only the vessel's wheelhouse (h> 8m above the water line) would remain visible. If we consult the Ship RCS Table [15], we will find a RCS of 1000 m² for the *bow view* of the most similar ship (coaster of 67 m length). In this one example it appears physically not impossible that just superstructure above the stated height represents the vessel's specified RCS.

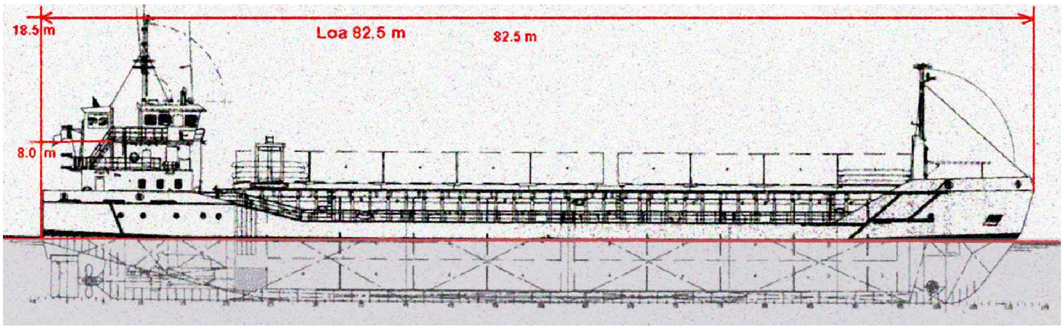


Figure 13: Example of an 82.5 m Cargo/Container Ship (2,301 gt)

Figure 14 below shows the impact of the height parameter on the range calculation. With a radar antenna height of 25 m the calculated radar horizon is 11 km. Up to this distance the entire superstructure of the ship (or the radar reflector at 8 m height) would be visible to the radar. If the distance to the radar is increased the ship/reflector would start dipping on the horizon. The remaining height as a function of the distance is shown by the graph in Figure 14. The ship/reflector has dipped the specification height of 8 m at a distance of 17.5 n.mi. Figure 1

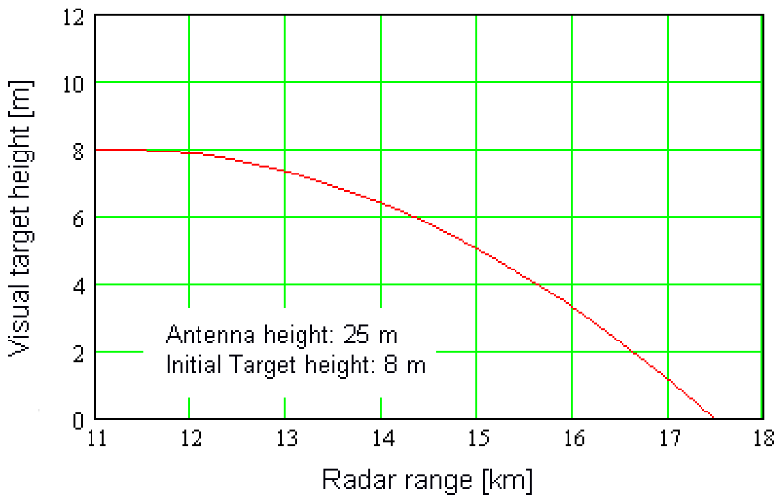


Figure 14: Visible Height of a Radar Target as a Function of Range

The range difference between 8 and 0 m target height is approx. 60%! This example shows the dimension of the uncertainty in using the height parameter in range calculations. The height parameter needs further explanation and should at present be used with caution.

10 Conclusions

This paper has elucidated the basic facts of the Radar Cross Section parameter, which are presently wide spread in difficult to obtain literature. These facts need to be known and correctly applied to obtain meaningful radar range calculation results. In order to make radar range calculations comparable it is suggested to:

- work out a standardised definition of RCS which contains the main factors that influence RCS specification, e.g. frequency, valid aspect/grazing arcs, Swerling case, level of statistical probability,
- compile a list of (measured and properly specified) RCS data for the most frequent types of ship.

The results of this work would not only be beneficial to foster general understanding of radar target detection, but would also facilitate the provision of concise and comparable radar range calculations. The scope of beneficiaries is quite wide, radar operators at sea or ashore, international organisations in charge for standardisation of maritime equipment, radar manufacturers, maritime safety organisations, and radar consultants.

11 References

[1] Barton/Leonov: *Radar Technology Encyclopedia*, Artech House

[2] Skolnik, *Introduction to Radar Systems*, McGraw-Hill

[3] *IEEE Standard Dictionary of Electrical and Electronic Terms,* ANSI/IEEE Std 100-1984

[4] Knott/Shaeffer/Tuley: *Radar cross Section*, Artech House

[5] www.mar-it.de/Radar/Blanket/blanket.htm

[6] Swerling, *Probability of Detection for Fluctuating Targets*, Rand Research Memorandum RM-1217, March 17, 1954

[7] Bowditch, *American Practical Navigator*, Vol. 1, 1977

[8] www.tno.nl/instit/fel/os/fac/carpet.html

[9] www.cadrcs.com

[10] IALA VTS Manual 3003, www.mar-it.de/Radar/RCS/IALA_RCS_Table.pdf

[11] Ingo Harre: *RCS in Range Prediction for Maritime Targets- Some Unresolved Questions*, International Radar Symposium, German Institute of Navigation (DGON), Dresden, 30 Sept. – 2 Oct. 2003.

[12] Williams/Cramp/Curtis: Experimental study of the radar cross section of maritime targets, Electronic Circuits and Systems, Vol. 2, No. 4,July 1978

[13] www.ussailing.org/safety/Studies/radar_reflector_test.htm

[14] F.V. Schultz, R.C. Burgener, S. King: Measurement of the Radar Cross Section of a Man, IRE Proceedings, Vol. 46, pp. 476-481, Feb. 1958

[15] www.mar-it.de/Radar/RCS/Ship_RCS_Table.pdf